

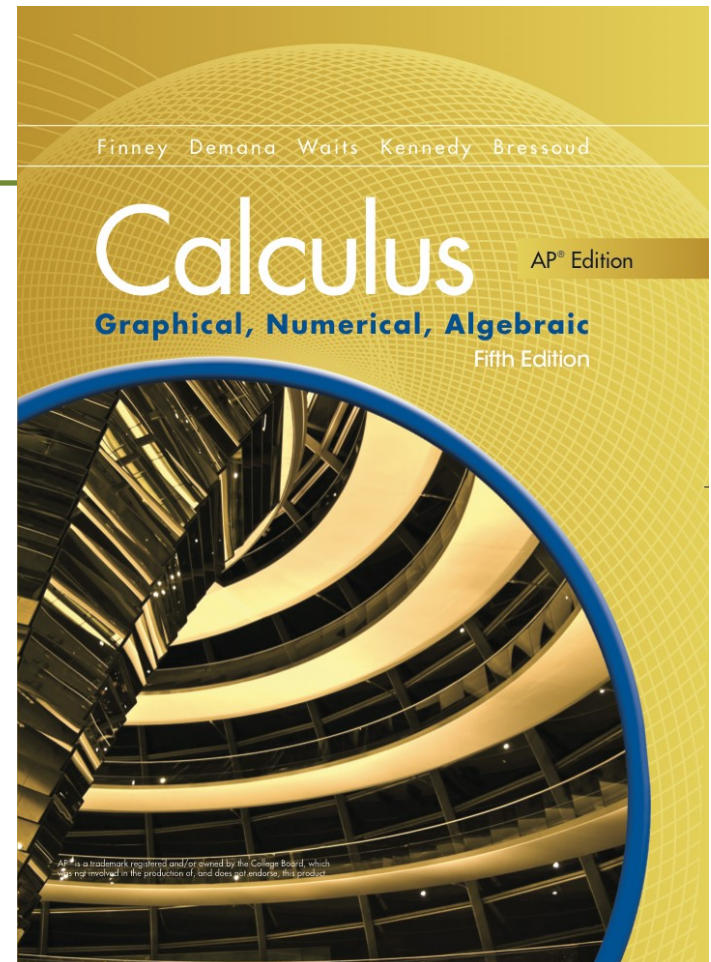
Chapter 6

Applications of Derivatives



Section 6.2

Definite Integrals



Quick Review

Evaluate the sum.

1. $\sum_{n=1}^4 n^2$

2. $\sum_{k=1}^4 (3k+1)$

Write the sum in sigma notation.

3. $2 + 3 + 4 + \dots + 49 + 50$

4. $2 + 4 + 6 + 8 + \dots + 98 + 100$

5. $3(1)^3 + 3(2)^3 + \dots + 3(100)^3$

6. Write the expression as a single sum in sigma notation $\sum_{n=1}^4 n^2 + 3\sum_{n=1}^4 n$

7. Find $\sum_{k=0}^n (-1)^k$ if n is odd.

8. Find $\sum_{k=0}^n (-1)^k$ if n is even.

Quick Review Solutions

Evaluate the sum.

1. $\sum_{n=1}^4 n^2$ 30

2. $\sum_{k=1}^4 (3k+1)$ 34

Write the sum in sigma notation.

3. $2+3+4+\dots+49+50$ $\sum_{k=2}^{50} k$

4. $2+4+6+8+\dots+98+100$ $\sum_{k=1}^{50} 2k$

5. $3(1)^3+3(2)^3+\dots+3(100)^3$ $\sum_{k=1}^{100} 3k^3$

6. Write the expression as a single sum in sigma notation $\sum_{n=1}^4 n^2 + 3\sum_{n=1}^4 n$ $\sum_{n=1}^4 (n^2 + 3n)$

7. Find $\sum_{k=0}^n (-1)^k$ if n is odd. 0

8. Find $\sum_{k=0}^n (-1)^k$ if n is even. 1

What you'll learn about

- Existence of definite integrals for continuous functions
- Terminology and notation of definite integration
- Definite integral as area
- Definite integral as accumulator
- Definite integrals with discontinuities

... and why

The definite integral is the basis of integral calculus, just as the derivative is the basis of differential calculus.

Sigma Notation

$$\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n$$

The Definite Integral as a Limit of Riemann Sums

Let f be a function defined on a closed interval $[a, b]$. For any partition P of $[a, b]$, let the numbers c_k be chosen arbitrarily in the subinterval $[x_{k-1}, x_k]$.

If there exists a number I such that
$$\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n f(c_k) \Delta x_k = I$$

no matter how P and the c_k 's are chosen, then f is **integrable** on $[a, b]$ and I is the **definite integral** of f over $[a, b]$.

The Existence of Definite Integrals

All continuous functions are integrable. That is, if a function f is continuous on an interval $[a, b]$, then its definite integral over $[a, b]$ exists.

The Definite Integral of a Continuous Function on $[a,b]$

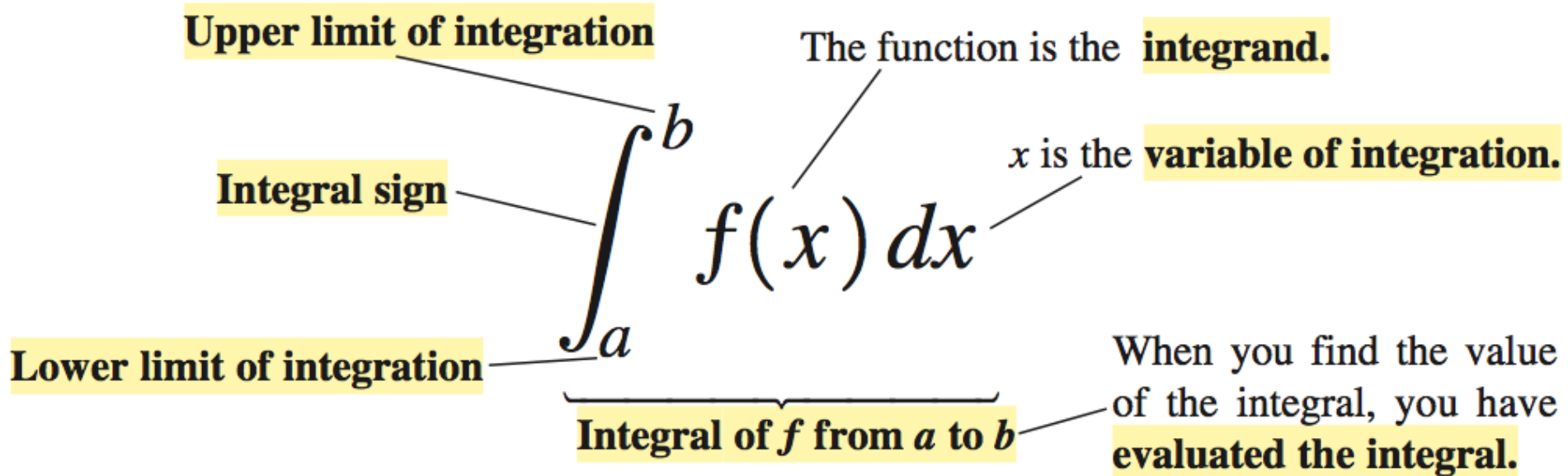
Let f be continuous on $[a,b]$, and let $[a,b]$ be partitioned into n subintervals of equal length $\Delta x = (b - a) / n$. Then the definite integral of f over $[a,b]$ is given by $\lim_{n \rightarrow \infty} \sum_{k=1}^n f(c_k) \Delta x$, where each c_k is chosen arbitrarily in the k^{th} subinterval.

The Definite Integral

$$\int_a^b f(x) dx$$

Read as “the integral from a to b of f of x dee x , or sometimes as “the integral from a to b of f of x with respect to x .”

The Definite Integral



Example Using the Notation

The interval $[-2, 4]$ is partitioned into n subintervals of equal length $\Delta x = 6/n$

Let m_k denote the midpoint of the k^{th} subinterval. Express the limit

$\lim_{n \rightarrow \infty} \sum_{k=1}^n (3(m_k)^2 - 2m_k + 5) \Delta x$ as an integral.

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n (3(m_k)^2 - 2m_k + 5) \Delta x = \int_{-2}^4 (3x^2 - 2x + 5) dx$$

Area Under a Curve (as a Definite Integral)

If $y = f(x)$ is nonnegative and integrable over a closed interval $[a, b]$, then the area under the curve $y = f(x)$ from a to b is the **integral**

of f from a to b $A = \int_a^b f(x) dx$

Area

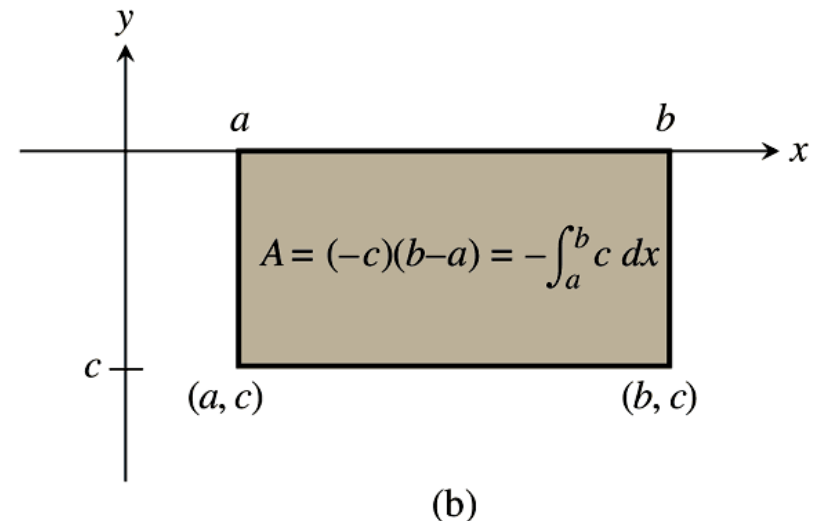
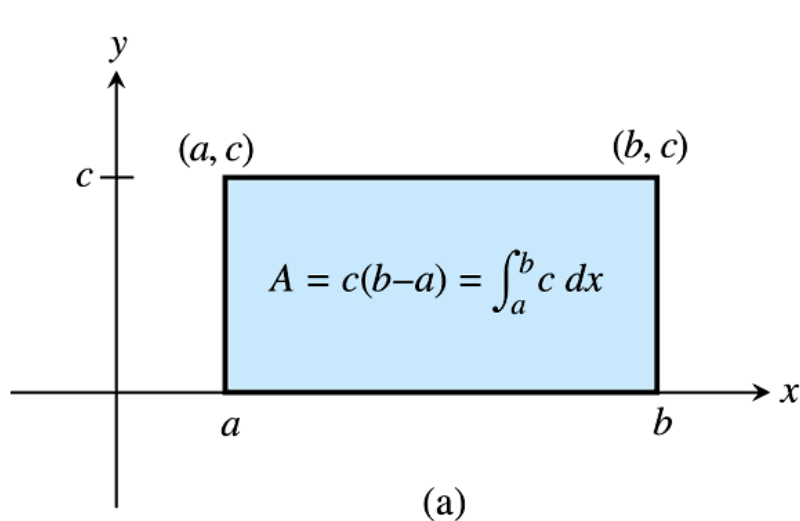
$$\text{Area} = - \int_a^b f(x) dx \text{ when } f(x) \leq 0.$$

$$\int_a^b f(x) dx = (\text{area above the } x\text{-axis}) - (\text{area below the } x\text{-axis}).$$

The Integral of a Constant

If $f(x) = c$, where c is a constant, on the interval $[a, b]$, then

$$\int_a^b f(x) dx = \int_a^b c dx = c(b - a)$$



Example Using NINT

Evaluate numerically. $\int_1^2 x \sin x dx$

$$\text{NINT}(x \sin x, x, -1, 2) \approx 2.04$$